

Towards Quantum Simulating QCD

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Abstract

Quantum link models provide an alternative non-perturbative formulation of Abelian and non-Abelian lattice gauge theories. They are ideally suited for quantum simulation, for example, using ultracold atoms in an optical lattice. This holds the promise to address currently unsolvable problems, such as the real-time and high-density dynamics of strongly interacting matter, first in toy-model gauge theories, and ultimately in QCD.

Keywords: Quantum simulation, sign problem, real-time dynamics of gauge theories

1. Introduction

Strongly interacting quantum matter may undergo intriguing real-time evolution. Prominent examples range from the expansion of an ultracold atomic gas released from a trap, or the out-of-equilibrium dynamics of a strongly correlated electron system, to an expanding quark-gluon plasma produced in a heavy-ion collision. Due to the enormous dimension of the Hilbert space, which increases exponentially with the system size, understanding the dynamics of large strongly coupled quantum systems is a notoriously hard problem. Even when considered in thermal equilibrium, Euclidean time Monte Carlo simulations of such systems often suffer from sign problems. For example, the fermionic Hubbard model away from half-filling or QCD at high quark density are currently inaccessible to first principles Monte Carlo simulations due to very severe fermion sign problems, thus preventing a better understanding of high-temperature superconductors or the dense cores of neutron stars. Real-time simulations of strongly interacting systems are also currently beyond reach, due to very severe complex weight problems in the corresponding real-time path integrals. Some sign problems even fall in the complexity class of NP-complete problems [1], which can be solved in polynomial time on a hypothetical “non-deterministic” computer, but for which no polynomial-time algorithm is known on an ordinary classical computer modeled by a Turing machine. Since one expects that $NP \neq P$ (where P is the complexity class of problems that are solvable in polynomial time on a classical computer), a generic solution for general sign problems is unlikely to exist. This does not exclude that specific sign problems may indeed be solvable on classical computers. In fact, several severe sign or complex action problems have been solved with the meron-cluster algorithm [2, 3, 4] or with the fermion bag approach [5, 6, 7]. Even the real-time evolution of a large strongly coupled quantum spin system, whose dynamics is entirely driven by measurements, has recently been simulated successfully with a cluster algorithm [8].

While it is not excluded that the doped Hubbard model or QCD at high quark density can be simulated on classical computers, it is already clear that a universal quantum computer could indeed overcome several of the limitations of classical computers [9, 10]. In particular, since it operates with quantum hardware, a quantum computer can naturally manipulate complex amplitudes and thus does not suffer from sign or complex weight problems. Although it is not known whether a quantum computer could solve NP-hard problems, it would be extremely useful for deepening our

understanding of the dynamics of strongly coupled quantum systems. While theoretical constructions for quantum computers based on ultracold trapped ions exist already for some time [11], it is difficult to predict when powerful quantum computers may become available. The D-wave devices based on a network of superconducting flux qubits have been used to operate the quantum adiabatic algorithm [12] on random instances of an Ising spin glass [13]. A comparison with simulated classical and quantum annealing algorithms led to the conclusion that the D-wave machines indeed perform quantum rather than classical annealing, but are not yet competitive with classical computers.

As early as 1982, Feynman proposed to use specifically designed quantum devices to simulate other quantum systems [14]. Based on the experimental breakthrough of realizing ultracold Bose-Einstein condensates [15, 16], Feynman’s vision of quantum simulators is becoming a reality. Although they are not universal quantum computers, but special purpose devices that are designed to mimic a specific quantum system, quantum simulators have the potential to drastically improve our understanding of strongly coupled quantum systems [17]. Quantum simulators have been constructed using ultracold atoms in optical lattices [18, 19], trapped ions [20], photons [21], or superconducting circuits on a chip [22]. One distinguishes digital [9] and analog [23] quantum simulators. A digital quantum simulator is a controllable many-body system, which encodes the state of the simulated system as quantum information, and executes a programmable sequence of quantum gate operations, following a stroboscopic Trotter decomposition, in order to represent its dynamics. Analog quantum simulators, on the other hand, realize continuous real-time evolution. They are limited to simpler interactions, but are more easily scalable to large system size. It was an experimental breakthrough when ultracold atoms in an optical lattice were used as an analog quantum simulator to address the quantum phase transition that separates a Mott insulator state (with localized particles) from a superfluid in the bosonic Hubbard model [24]. The optical lattice is realized with counter-propagating laser beams, whose intensity determines the amplitude for hopping between neighboring lattice sites. The quantum simulation has been validated in comparison with accurate quantum Monte Carlo simulations [25], which are possible because the Bose-Hubbard model does not suffer from a sign problem. The next challenge in this field is to quantum simulate the fermionic Hubbard model, in order to decide whether it can describe high-temperature superconductivity. This requires reaching even lower temperatures in the quantum simulator than the ones achieved until now.

When we use a quantum simulator to learn more about the dynamics of a strongly coupled system, we are actually doing an experiment. As usual in quantum physics, the device is first prepared in an initial state, it then evolves in time driven by its Hamiltonian, and finally it is analyzed by measurements. By repeating the quantum simulation experiment many times, and by averaging over the measurement results, one can compare it with the predictions of quantum mechanics. Successful quantum simulations benefit from close collaboration between theorists and experimentalists, from appropriately designing to accurately performing and theoretically validating the experiments. Since quantum simulators are currently far from being standard devices, a theorist will want to know whether they are an appropriate tool for theoretical physics. Ultimately, this will depend on their accuracy and reliability, which is currently limited but should improve significantly with time. Certainly, classical computers have matured to the point where a theorist performing, for example, a Monte Carlo simulation does not necessarily think of herself as an experimentalist, despite the fact that she is operating a very sophisticated instrument. Instead, she may think of a classical computer as a “black box” that perfectly realizes the computational model of a Turing machine, which just extends our mathematical capabilities. It is currently unclear when (or even whether) quantum simulators or quantum computers will reach a similar status, but there is no reason to be overly pessimistic. Even our brain, which is likely the most sophisticated piece of “hardware” in the universe, is not always completely reliable, but a theorist is not reluctant to use it as best as possible. Quantum simulators and ultimately quantum computers promise to significantly extend our brain’s capabilities in qualitatively new directions, and we should not be reluctant to push their development forward as much as possible.

Proposals for quantum simulator constructions already exist for some simple bosonic [26, 27, 28] and fermionic [29, 30, 31, 32] field theories. Hence it is natural to ask whether our understanding of strongly coupled systems in nuclear and particle physics may benefit from quantum simulation. This certainly is a long-term project, which can only be realized in small steps. Before one addresses full QCD, one should gain experience with simpler toy-model gauge theories. Such models may exist in a lower space-time dimension, they may have a simpler $U(1)$ or $SU(2)$ gauge group, instead of QCD’s $SU(3)$ gauge symmetry, or they may exist on a lattice away from the continuum limit. They may also have a reduced $\mathbb{Z}(2)$ instead of Nature’s $SU(2)_L \times SU(2)_R$ chiral symmetry, or they may have bosonic instead of fermionic baryons. All this implies that, for quite some time, we should not expect quantum simulations to yield quantitative results that are directly relevant to nuclear or particle physics. Still, the toy-models mentioned

above share different qualitative features with QCD, and will have similar dynamics, at least in some respects. In particular, like QCD, simple toy-model gauge theories may have a spontaneously broken chiral symmetry, which is restored at finite baryon density. They may also display color superconductivity at high baryon density, with or without color-flavor locking. One can also imagine that a gauge theory quantum simulator can mimic qualitative features of heavy-ion collisions. Since we have no other way of reliably investigating QCD's real-time dynamics from first principles, the construction of quantum simulators for Abelian and non-Abelian gauge theories, with or without fermionic matter, is timely and most promising [33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48]. Once such systems are realized in the laboratory, they will also become interesting objects of study in their own right.

If quantum simulations were ultimately limited to toy-model gauge theories, they would be of limited interest for nuclear and particle physics. While the first steps towards gauge theory quantum simulations will be taken in simpler gauge theories, quantum link models provide a long-term vision for how to eventually realize quantum simulations of QCD. The first $U(1)$ and $SU(2)$ gauge theories formulated with (generalized) quantum spins were constructed by Horn [49], and further investigated by Orland and Rohrlich under the name of gauge magnets [50]. Quantum link models [51, 52, 53] provide an alternative to Wilson's lattice gauge theory [54, 55, 56] for defining QCD beyond perturbation theory. Both in the Wilson formulation and in quantum link models, the basic gauge degrees of freedom are associated with the links connecting nearest-neighbor sites of a lattice that serves as a regulator of ultra-violet divergences. In Wilson's lattice gauge theory, the link variables are parallel transporter matrices that take values in the gauge group. In particular, in lattice QCD they are $SU(3)$ matrices associated with each link. Since the $SU(3)$ group space is a continuous manifold, in the Wilson theory the local link Hilbert space is infinite-dimensional. This poses a severe challenge for quantum simulations with ultracold atoms, because in practice only a small number of individual atomic states can be used to encode quantum information. In contrast to Wilson's lattice gauge theory, quantum link models are ideally suited for quantum simulation because they have a finite-dimensional Hilbert space per link. While $SU(N)$ quantum links are still $N \times N$ matrices, unlike in the Wilson theory, their matrix elements are non-commuting operators, which are generators of an $SU(2N)$ embedding algebra. In the quantum link formulation of QCD, the gauge degrees of freedom are described by a 20-dimensional representation of an $SU(6)$ embedding algebra. Hence five qubits (with a $2^5 = 32$ -dimensional Hilbert space) are more than one needs to represent the quantum information encoded in an $SU(3)$ quantum link, while a Wilson-type link variable exists in an infinite-dimensional Hilbert space. Thanks to universality, QCD can be regularized in several different ways, and it is useful to choose the most appropriate regularization, depending on the question one wants to address. Certainly, when one wants to study QCD at very high energy, thanks to asymptotic freedom, one can use perturbation theory, which is best handled using dimensional regularization and renormalization applied to the standard QCD Lagrangian. On the other hand, when one wants to address non-perturbative questions concerning, for example, static properties of hadrons, Wilson's lattice QCD has proved to be the best suited theoretical framework. Here we argue that the quantum link formulation of QCD is ideally suited for quantum simulations that address the real-time dynamics of quarks and gluons as well as QCD at high quark density.

In Sections 2 and 3, we will discuss quantum simulators for Abelian and non-Abelian quantum link models, which share qualitative features with QCD. The concluding Section 4 outlines what needs to be done in order to ultimately approach quantum simulations of QCD itself.

2. Quantum Simulators for Abelian Gauge Theories

Quantum simulators encode the state of the simulated quantum system as quantum information in its discrete quantum states. Ultracold atomic gases in optical lattices are used as quantum simulators for condensed matter systems including bosonic or fermionic Hubbard models. The information about the corresponding quantum states is then encoded in the discrete positions of fermionic or bosonic atoms in the wells of the optical lattice as well as in the discrete internal states of the atoms. The gauge theories of particle physics, including QCD, are usually formulated with continuous gauge field degrees of freedom. Even in Wilson's lattice QCD, in which the gauge field resides on the links of a discrete lattice, the link variables themselves are continuous variables which give rise to an infinite-dimensional link Hilbert space. It is not clear how Wilson's continuously varying link variables, which take values in the gauge group ($SU(3)$ for QCD), could be represented with a handful of discrete states, for example, of ultracold atoms. Quantum link models provide an alternative lattice regularization of gauge field theories including QCD [52],

in which the link Hilbert space is finite-dimensional, and can thus be naturally represented, for example, with discrete states of ultracold atoms in an optical lattice.

In order to introduce quantum link models in a simple context, we first consider a $U(1)$ gauge theory. Quantum link models share most of the structure of Wilson's lattice gauge theory in the Hamiltonian formulation, in which space is replaced by a cubic lattice and time remains continuous. In this formulation, the gauge field is represented by classical parallel transporter link variables $U_{xy} = \exp(i\varphi_{xy}) \in U(1)$, which take values in the gauge group. Their canonically conjugate momenta are electric field operators $E_{xy} = -i\partial/\partial\varphi_{xy}$, which are again associated with the links, and which obey the commutation relations

$$[E_{xy}, U_{x'y'}] = \delta_{xx'}\delta_{yy'}U_{xy}, \quad [E_{xy}, U_{x'y'}^\dagger] = -\delta_{xx'}\delta_{yy'}U_{xy}^\dagger, \quad (1)$$

such that operators associated with different links commute with each other. Also introducing staggered fermion creation and annihilation operators ψ_x^\dagger, ψ_x , which obey canonical anti-commutation relations

$$\{\psi_x^\dagger, \psi_y\} = \delta_{xy}, \quad \{\psi_x^\dagger, \psi_y^\dagger\} = \{\psi_x, \psi_y\} = 0, \quad (2)$$

the Hamiltonian of lattice QED takes the form

$$H_{\text{QED}} = -t \sum_{\langle xy \rangle} s_{xy} (\psi_x^\dagger U_{xy} \psi_y + \psi_y^\dagger U_{xy}^\dagger \psi_x) + m \sum_x s_x \psi_x^\dagger \psi_x + \frac{e^2}{2} \sum_{\langle xy \rangle} E_{xy}^2 - \frac{1}{4e^2} \sum_{\square} (U_{\square} + U_{\square}^\dagger). \quad (3)$$

Here t is a hopping parameter, m is the fermion mass, and e is the electric charge. The factor $s_x = (-1)^{x_1 + \dots + x_d}$ is associated with the sites and the factor s_{xy} is associated with the links $\langle xy \rangle$ connecting nearest-neighbor sites x and y of the d -dimensional spatial lattice. The links in the 1-direction have $s_{xy} = 1$, the ones in the 2-direction have $s_{xy} = (-1)^{x_1}$, and those in the k -direction have $s_{xy} = (-1)^{x_1 + \dots + x_{k-1}}$. The product $U_{\square} = U_{wx} U_{xy} U_{yz} U_{zw}$ around an elementary lattice plaquette \square represents the magnetic field energy. The Hamilton operator is gauge invariant, i.e. it commutes with the generators of local gauge transformations

$$G_x = \psi_x^\dagger \psi_x + \sum_k (E_{x, x+\hat{k}} - E_{x-\hat{k}, x}), \quad [H, G_x] = 0, \quad (4)$$

where \hat{k} is a unit-vector in the k -direction. The gauge generators G_x represent the difference of the local charge density $\psi_x^\dagger \psi_x$ and the lattice divergence of the electric field. Physical states $|\Psi\rangle$ must obey the Gauss law $G_x|\Psi\rangle = 0$, i.e. they must be gauge invariant. Since U_{xy} is a continuously varying link variable, $E_{xy} \in \mathbb{Z}$ can take infinitely many values, and thus the Hilbert space of Wilson's lattice gauge theory is infinite-dimensional, already for a single link.

Quantum link models realize continuous gauge symmetries with discrete gauge variables and thus in a finite-dimensional link Hilbert space. In an Abelian $U(1)$ quantum link model, the link operator as well as the electric field operator are given by quantum spin operators \vec{S}_{xy} associated with the link xy

$$U_{xy} = S_{xy}^1 + iS_{xy}^2 = S_{xy}^+, \quad U_{xy}^\dagger = S_{xy}^1 - iS_{xy}^2 = S_{xy}^-, \quad E_{xy} = S_{xy}^3. \quad (5)$$

The quantum link operators U_{xy} and U_{xy}^\dagger act as raising and lowering operators of electric flux. The eqs.(1), (3), and (4) of the Wilson theory still hold unchanged in quantum link models. The only difference, which allows the drastic simplification of a finite-dimensional Hilbert space per link, is that now $[U_{xy}, U_{x'y'}^\dagger] = 2\delta_{xx'}\delta_{yy'}E_{xy} \neq 0$.

Although the simple $(2+1)$ -d $U(1)$ quantum link model with a quantum spin $\frac{1}{2}$ is not completely equivalent to the Wilson theory, it shares the important feature of confinement. The dynamics of this model is illustrated in Fig.1. Both digital and analog quantum simulator constructions have been proposed for this system. Highly excited Rydberg atoms in an optical lattice, which can be addressed individually with external lasers, have strong long-range dipole-dipole interactions, thus allowing the entanglement of a number of atoms with a single control atom. This forms the basis of digital quantum simulator constructions for $U(1)$ quantum link models [34, 43]. These use control atoms at lattice sites to ensure Gauss' law, and at plaquette centers to flip electric flux loops, while other Rydberg atoms act as qubits that represent the quantum link variables. Some analog quantum simulator constructions again use ultracold atoms in optical lattices [33, 36, 39], while others are based on superconducting circuits on a chip [46, 47].

In [40, 42, 45] Bose-Fermi mixtures were proposed to quantum simulate $U(1)$ quantum links coupled to dynamical staggered fermions. Then the string that connects external static charges can break by the creation of a dynamical charge-anti-charge pair. As illustrated in Fig.2, the proposed quantum simulator can investigate string-breaking in real time.

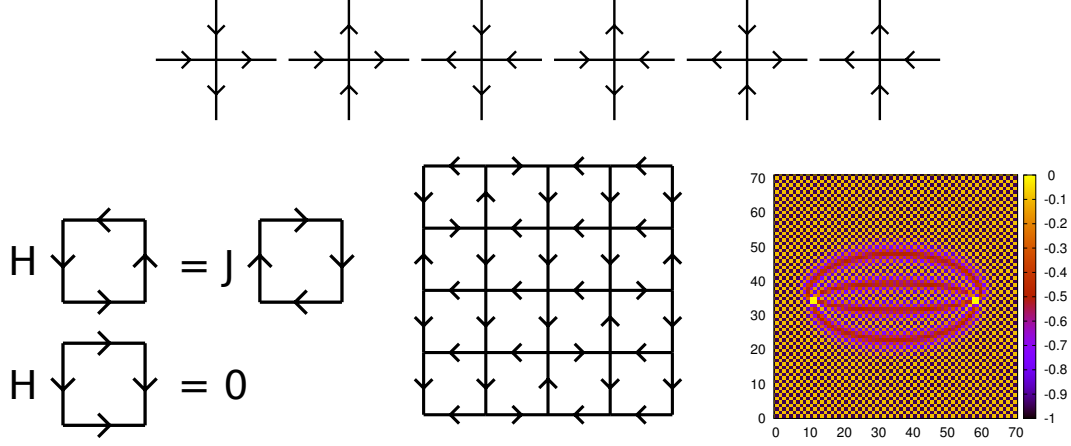


Figure 1. Top: The six flux configurations that satisfy the Gauss law in the $(2+1)$ -d $U(1)$ quantum link model. Bottom left: The Hamiltonian reverses the direction of a closed loop of flux, and annihilates all other flux states. Bottom middle: A flux configuration on a 4×4 lattice with periodic boundary conditions, with the fluxes obeying Gauss' law at every site x . Bottom right: Energy density $-(U_{\square} + U_{\square}^{\dagger})$ of the string connecting two external charges $Q = \pm 2$. The string splits into four strands, each carrying electric flux $\frac{1}{2}$ [57].

3. Quantum Simulators for non-Abelian Gauge Theories

Non-Abelian gauge theories can also be regularized with quantum link models. Like a Wilson-type $SU(N)$ link variable, a quantum link in an $SU(N)$ gauge theory still is an $N \times N$ matrix. However, its matrix elements U_{xy}^{ij} are no longer complex numbers, but non-commuting operators. Together with non-Abelian electric flux operators L_{xy}^a and R_{xy}^a residing on the left and right end of a link, and an Abelian electric flux E_{xy} , the quantum link operators form the embedding algebra $SU(2N)$. The $2N^2$ real and imaginary parts of U_{xy}^{ij} , together with the $2(N^2 - 1)$ $SU(N)$ generators L_{xy}^a and R_{xy}^a and the generator E_{xy} , indeed provide the $2N^2 + 2(N^2 - 1) + 1 = 4N^2 - 1$ generators of $SU(2N)$. Quantum link models can realize exact continuous $SU(N)$ gauge symmetry with any finite-dimensional representation of $SU(2N)$, while the Wilson theory again has an infinite-dimensional Hilbert space per link. In particular, $SU(3)$ quantum link QCD uses a 20-dimensional representation of the embedding algebra $SU(6)$. Just as in the standard Wilson formulation, in the quantum link regularization of QCD the Hamiltonian is given by

$$H_{\text{QCD}} = -t \sum_{\langle xy \rangle} s_{xy} (\psi_x^{\dagger} U_{xy} \psi_y + \psi_y^{\dagger} U_{xy}^{\dagger} \psi_x) + m \sum_x s_x \psi_x^{\dagger} \psi_x + \frac{g^2}{2} \sum_{\langle xy \rangle} (L_{xy}^2 + R_{xy}^2) - \frac{1}{4g^2} \sum_{\square} \text{Tr}(U_{\square} + U_{\square}^{\dagger}), \quad (6)$$

which commutes with the infinitesimal generators of gauge transformations

$$G_x^a = \psi_x^{\dagger} \lambda_{ij}^a \psi_x^j + \sum_k (L_{x,x+\hat{k}}^a + R_{x-\hat{k},x}^a), \quad [G_x^a, G_y^b] = 2i\delta_{xy} f_{abc} G_x^c. \quad (7)$$

It is a remarkable feature of quantum link models, that the gauge degrees of freedom can be expressed as fermion bilinears

$$L_{xy}^a = c_{x,+}^{\dagger} \lambda_{ij}^a c_{x,+}^j, \quad R_{xy}^a = c_{y,-}^{\dagger} \lambda_{ij}^a c_{y,-}^j, \quad E_{xy} = \frac{1}{2} (c_{y,-}^{\dagger} c_{y,-}^i - c_{x,+}^{\dagger} c_{x,+}^i), \quad U_{xy}^{ij} = c_{x,+}^i c_{y,-}^{j\dagger}. \quad (8)$$

The fermionic constituents of the gauge field are called rishons and obey standard anti-commutation relations. The rishon dynamics is illustrated in Fig.3.

A digital quantum simulator construction for an $SU(2)$ pure gauge theory using Rydberg atoms in an optical lattice has been proposed in [43]. An analog quantum simulator for an $SU(2)$ gauge theory coupled to fermionic matter using a Fermi-Bose mixture of atoms in an optical superlattice has been constructed theoretically in [44, 45]. An alternative analog proposal embodies the rishons of quantum link models with fermionic ^{87}Sr or ^{173}Yb alkaline-earth atoms moving in an optical superlattice (c.f. Fig.3) [41]. The interactions between the atoms can be engineered

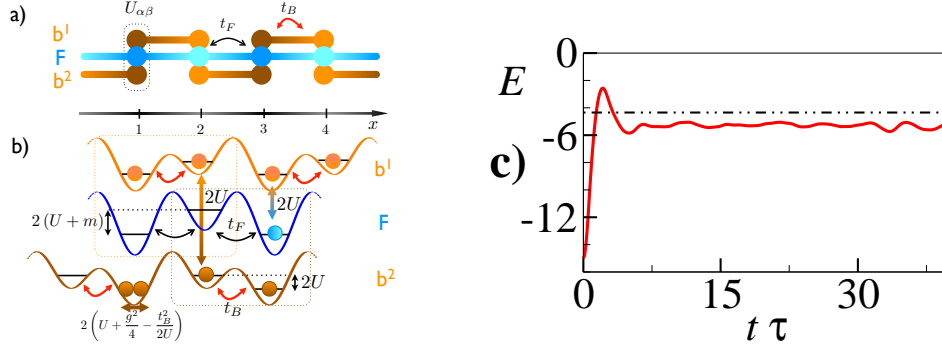


Figure 2. a) Quantum simulator of a $(1+1)$ -d $U(1)$ quantum link model with staggered fermion matter based on a Bose-Fermi mixture of ultracold atoms hopping in a 3-strand optical superlattice. b) Mass m , hopping t , and interaction U , g parameters are determined by the shape of the optical superlattice. Energy conservation favors the simultaneous hopping of fermions and bosons, thus ensuring gauge invariance. c) Dynamical string breaking in real time: the string energy is converted into the mass of a charge-anti-charge pair, thus depleting the electric flux E of the string [40].

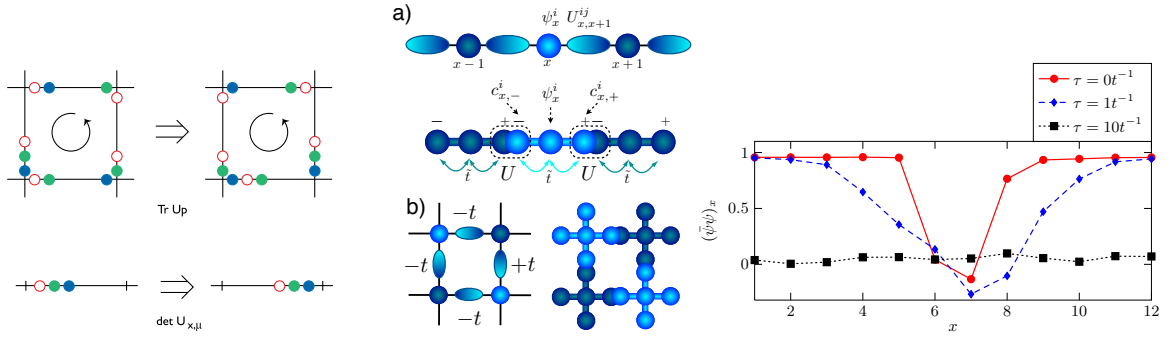


Figure 3. Left: In an $SU(3)$ quantum link model, three fermionic rishons of different colors reside on each link. The Hamiltonian moves the rishons around plaquettes or along links, like the beads of an abacus. Middle: Quantum simulator for an $SU(N)$ gauge theory with ultracold alkaline-earth atoms representing “quarks” or rishon constituents of “gluons” in a 1-d (a) or 2-d (b) optical superlattice. Depending on its position in the optical lattice, an alkaline-earth atom either embodies a “quark” or a rishon. Gauge invariance is protected by the internal $SU(2I+1)$ symmetry of the nuclear spin I of the atoms. Right: Real-time τ evolution of the chiral order parameter profile $(\bar{\psi}\psi)_x$ in a $U(2)$ gauge theory, mimicking the expansion of a hot “quark-gluon” plasma [41].

using Feshbach resonances. The $SU(N)$ color of a rishon is encoded in the Zeeman states of the nuclear spin I , which have a remarkably precise $SU(2I+1)$ symmetry that naturally protects $SU(N)$ gauge invariance. Each link has two rishon-sites, one at each end. In addition, there are the ordinary lattice sites where “quarks” reside. When an alkaline-earth atom sits on a rishon-site it embodies a fermionic constituent of a “gluon”. The same atom represents a “quark” when it hops to an adjacent ordinary lattice site. Even a simple $U(2)$ toy-model gauge theory, which can be realized with a single rishon per link, is quite interesting, because it has a spontaneously broken chiral symmetry, whose dynamics can be investigated in real time (c.f. Fig.3).

The simple $U(2)$ toy-model gauge theory does not have “baryons”, because baryon number is part of the gauge symmetry. Another toy-model gauge theory that does contain “baryons” and thus can be used to mimic qualitative features of nuclear physics has an $SO(3)$ gauge symmetry. When one puts “quarks” in the adjoint triplet representation of $SO(3)$, such that they have the same color quantum numbers as $SO(3)$ “gluons”, one can form color-singlet “baryons” that consist of a single “quark” confined to a “gluon”. Such a system can be quantum simulated using magnetic atoms with dipole-dipole interactions in an optical lattice [58]. Even a simple $(1+1)$ -d toy-model $SO(3)$ gauge theory has a $\mathbb{Z}(2)$ chiral symmetry which is spontaneously broken in the vacuum, but restored at finite baryon density. Fig.4 shows the energy difference ΔE between the vacuum state and the first excited state as a function of the spatial volume L .

At zero baryon density n_B , ΔE decreases exponentially with L , thus indicating vacuum degeneracy, and hence spontaneous $\mathbb{Z}(2)$ chiral symmetry breaking, in the infinite-volume limit. At higher baryon densities, $n_B \geq \frac{1}{2}$, ΔE no longer decreases exponentially, thus indicating that chiral symmetry gets restored. A quantum simulator could be used to investigate the high baryon-density phase diagram of this toy-model, which may even show color-superconductivity.

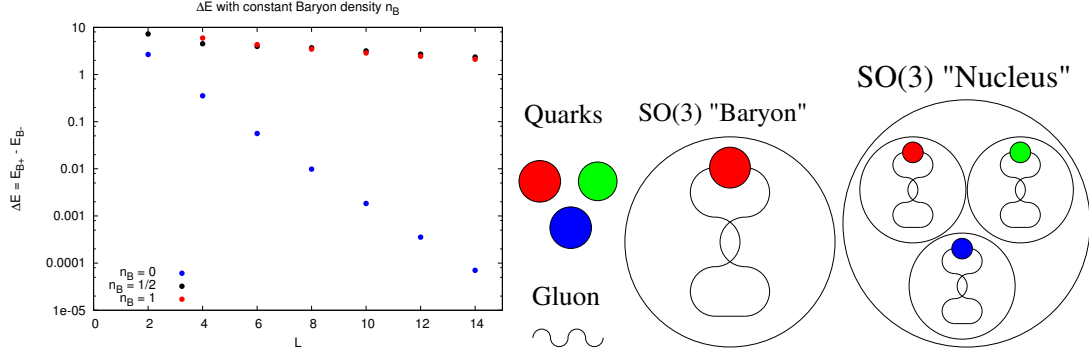


Figure 4. Left: Chiral symmetry restoration in a $(1+1)$ -d $SO(3)$ toy-model gauge theory with adjoint fermions. Right: Adjoint triplet “quarks” and “gluons” in a toy-model $SO(3)$ gauge theory. An $SO(3)$ “baryon” consists of a single triplet “quark” and a triplet “gluon” forming a color singlet. Several baryons may form an $SO(3)$ “nucleus”.

4. Conclusions

Quantum link models provide an alternative non-perturbative regularization of lattice gauge theories, which is ideally suited for quantum simulation, for example, using ultracold atomic gases in optical lattices. This holds the promise to address very challenging problems, such as the real-time and high density dynamics of strongly interacting matter, which, due to very severe sign or complex action problems, cannot be addressed with classical computers. While it is hard to predict when quantum simulations of full QCD might become feasible, toy-model gauge theories that share qualitative features with QCD should be possible to quantum simulate within the next few years. In this way one can gain qualitatively new insights into highly non-trivial dynamics of Abelian and non-Abelian gauge theories, such as string breaking in real time, the expansion of a chirally restored “fireball”, or the restoration of chiral symmetry at high “baryon” density. If such quantum simulations can be performed successfully, in the long run quantum simulators may advance to a more and more accepted tool of theoretical physics. Before universal quantum computers become available, one may thus hope to eventually develop a special purpose quantum simulator to address currently unsolvable problems in QCD. The quantum link formulation of QCD provides a theoretical framework as well as a vision how the path towards this long-term goal could look like. Along this path, one must overcome severe challenges: proceeding to higher dimensions, realizing the full chiral symmetry of light quarks and the correct gauge group $SU(3)$, and, finally, taking the continuum limit. None of these difficult tasks seem insurmountable, at least in principle. While the ultimate goal of this research may be in the distant future, starting to work in this direction seems timely now. Interdisciplinary research between quantum optics, atomic, and particle physics, along the lines suggested here, holds the promise to deepen our understanding of the fundamental gauge structures underlying all of physics. There is a lot of exciting physics to be explored along the way towards a full QCD quantum simulator as a reliable tool for nuclear and particle physics.

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